NATIONAL ADVISORY COMMITTEE
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TECHNICAL NOTE

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FOR REFERENCE

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AN ELECTRICAL COMPUTER FOR THE SOLUTION OF SHEAR-LAG

AND BOLTED-JOINT PROBLEMS

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SUMMARY

The analogy between the distribution of stresses in flat
stiffened panels and the distribution of electric current in a
ladder-type resistance network is used as the theoretical basis
of an electrical computer for the rapid solution of shear-lag
problems. The computer, consisting of variable resistors and
multiple-current sources, is described; and typical examples are
given of its use. The analogy is extended to include bolted-
joint problems, and an example is given also.

INTRODUCTION

When a load is applied along the flange at the edge of a
flat stiffened panel (fig. 1), some of the load is transmitted
to the various stiffeners through the medium of the sheet, which
is placed in shear by the action. Similarly, when two plates
are connected by a bolted strap (fig. 2), the load is transmitted
from the plates to the strap through the medium of the bolts. As
shown in reference 1 and in appendix A, an analogy can be drawn
between the distribution of forces in the cases previously
described and the distribution of electric current in a ladder
network. This analogy is very useful in obtaining a numerical
solution, particularly in complicated cases which are not readily
amenable to mathematical solution. The time saved in such cases
is of appreciable importance.

In reference 1 the solution of a shear-lag problem was
obtained by means of an analogous network consisting of properly
chosen fixed resistors. The power supply consisted of a single
battery; the proper input currents were obtained from a voltage
divider by trial and error.

The present paper describes a pilot model of a more elaborate
setup. The model uses variable resistors and several sources of

current, both of which can be introduced independently into the network. Being readily adaptable to the solution of more than one problem, this construction converts the apparatus into a computer.

SYMBOLS

A   cross-sectional area, square inches

B   load carried by an individual bolt, pounds

C   bolt constant, inches per kip

D   diameter of bolt, inches

E   Young's modulus, ksi

G   shear modulus, ksi

I   longitudinal current, milliamperes

K   axial flexibility, inches per kip

L   axial length, inches

P   applied load, pounds

R   longitudinal resistance, ohms

S   shear force, pounds

V   electric potential, millivolts

b   transverse width, inches

h   depth of beam, inches

i   current in transverse resistors, milliamperes

p   pitch, inches

r   resistance of transverse resistors, ohms

t   thickness, inches

u　　　　axial displacement, inches

α　　　　proportionality factor for resistance

β　　　　proportionality factor for load

σ　　　　direct (normal) stress, psi

τ　　　　shear stress, psi

Subscripts:

b　　　　bolt

F　　　　flange

L　　　　longitudinal (stringer)

m　　　　a particular transverse row of resistors or bolts

n　　　　any transverse row of resistors or bolts

p　　　　plate

s　　　　sheet or strap

T　　　　total

av　　　average

DESCRIPTION OF COMPUTER

　　　　The front panel of the computer supports the variable
resistors constituting the network, together with the resistor
control knobs and dials, as shown in figure 3. In order to set
any element to a given resistance, the dial is turned to the
proper reading as determined by the calibration constant for
that element since the dials are graduated in 100 arbitrary
divisions rather than in ohms. The dial constants were individually
determined; consequently, it was possible to set the resistance
elements to ±1 percent or ±2 ohms, whichever unit was the greater.

　　　　The resistor elements are connected to form a ladder-type
network as shown in the wiring diagram in figure 4. Each element
has a maximum and minimum resistance of approximately 1000 ohms

and 10 ohms, respectively. Turning the dial to the extreme low
end short-circuits the minimum resistance of the element; whereas
turning the dial to the high end open-circuits the element after
the 1000-ohm limit is reached.

Current may be independently introduced into the network at
eight points along the two sides opposite the ground connections.
The current is supplied by electronic regulating circuits (fig. 5),
which are designed to give negligible reaction upon each other
and to be independent of the resistance setting of the network.
The effect of line-voltage variation is also negligible. The
output of the regulating circuits can be varied from about 1
to 15 milliamperes by means of control dials on the front of the
panel. Figure 6 is a rear view of the computer and shows the
arrangement of the resistors and the current supplies.

The current in any element, as well as the output of the
electronic supply units, is determined by plugging in a meter
to a current-measuring jack associated with the desired element.
A high-grade milliammeter is used and allows the current to be
measured with an accuracy of about 1/4 percent of full scale.

The chief sources of error in the apparatus itself arise from
inaccuracy in resistor values and in the current measurement. In
the present model, an attempt was made to make a reasonable com-
promise between absolute minimization of all sources of error on
one hand and cost and convenience on the other hand. For instance,
the variable resistors are of comparatively high resistance; thus,
the errors due to variable contact resistance and meter-insertion
loss are reduced. If the apparatus is allowed to warm up for
about 15 minutes, errors due to further heating are negligible.

The errors inherent in the method, when used for shear-lag
problems, are due to the division of the structure into a finite
number of bays (corresponding to an electrical transmission line
with lumped constants) and due to the inclusion of the sheet area
with the stringer area to account for the axial stress in the
sheet. These errors have been discussed and evaluated in
reference 1.

ANALOGY FOR SHEAR-LAG PROBLEMS

As shown in reference 1 the correspondence between the
mechanical structure and the electrical network is given in the

notation of the present paper by the following table:

Structure		Network	
Quantity	Symbol	Quantity	Symbol
Axial load	P	Longitudinal current	I
Axial flexibility	$K = \dfrac{L}{EA}$	Longitudinal resistance	R
Shear force	S	Transverse current	i
Shear flexibility	$\dfrac{b}{GtL}$	Transverse resistance	r
Axial displacement	u	Potential	V

Written in equation form with arbitrary constants β and α, the following relations hold:

$$I = \beta P = \beta \sigma A \tag{1}$$

$$R = \alpha \frac{L}{EA} \tag{2}$$

$$i = \beta S = \beta \tau L t \tag{3}$$

$$r = \alpha \frac{b}{GtL} \tag{4}$$

$$V = \alpha \beta u \tag{5}$$

The boundary conditions are not included in these equations and, therefore, have to be determined from the individual problem, in which current corresponds to force and potential differences correspond to displacements. At a fixed boundary no displacement occurs; hence, there must be no potential difference along that part of the network representing the boundary. The electrical resistance corresponding to a fixed edge is therefore zero, which means that any amount of current may flow without producing a potential difference.

It is interesting to note that at a loaded edge or point the force against displacement characteristics of the applied load may vary from a dead weight, where the force is independent of any motion of the end of the specimen, to a screw-type loading machine, where the applied force varies greatly with slight motions of the specimen. Between these two extremes would be a structure which is loaded by another, the stiffness of which is of a corresponding order of magnitude.

Electrically, the preceding loading conditions correspond to
current supplied by a constant-current generator, a constant-
voltage generator, and a generator with an internal resistance
intermediate between infinity and zero. The present computer is
built with constant-current generators and, therefore, obtains
solutions which apply to the dead-weight type of loading since
this type of loading agrees more closely with the loading of the
actual structure.

The differences between the computer solution, wherein the
structure is broken up into a finite number of sections, and the
exact solution, wherein the number of sections is increased
without limit, should be considered.

In figure 7 a simple panel is shown loaded at one end of the
flange. The axial force in the flange gradually "leaks off" as
shear force into the sheet; the shear force in turn is transmitted
as axial force into the stringer. The amount of force leaking off
will be determined by the shear stiffness of the sheet and the
axial stiffnesses of the flange and stringers. The smooth curve
shown in figure 7 thus represents the stress in the flange. The
curve is high at the loaded end and decreases smoothly as the
distance from the end increases.

In order to set up the equivalent electrical network, the
structure must be broken into a finite number of sections so that
a section of resistor network may be adjusted to correspond to
each section of the structure. As the distance increases uniformly
along the electrical network, the current in the resistors
representing the flange remains constant in any one resistor and
then abruptly decreases to the next lower value as a junction
point is reached. This condition is graphically illustrated by
the series of steps in figure 7. At the points where the step
function crosses the smooth curve there is, of course, no error
due to the finite steps. At all other points a relative error
will exist between the two curves; this error may generally be
reduced in magnitude by adjusting the steps to lie half above
and half below the smooth curve.

In order to approach this condition, the stiffness of each
section is considered to lie at its center, and the sections are
shifted so that their centers come over the points L_0, L_1, and
L_2 in figure 7. The first section must therefore be made of
half length with the stiffness concentrated at the loaded end;
the last section will also be of half length but with the stiffness
concentrated at the point L_3, or root. Since the end conditions
fix the shear at the root as zero, this last half section does not

enter into the calculation. When the stiffnesses are adjusted in this manner, a smooth curve drawn through the midpoint of each step will closely approach the exact solution..

NUMERICAL EXAMPLES OF SHEAR-LAG PROBLEMS

Analysis of a Sheet-Stringer Panel

As an illustrative example the panel shown in figure 8 was chosen for analysis.. A single-stringer structure was used in order to compare results with the exact solution as obtained by the method given in appendix B of reference 2.

As shown in figure 8, the numerical values of the symbols are

$$b = 12 \text{ inches}$$

$$t = 0.024 \text{ inch}$$

$$A_F = 0.228 \text{ square inch}$$

$$A_L = 0.500 \text{ square inch}$$

$$L = 100 \text{ inches}$$

Also

$$E = 10,400 \text{ ksi}$$

$$G = 4000 \text{ ksi}$$

In order to agree with the configuration of the computer network, the structure was divided lengthwise into five equal parts. Substituting the foregoing values in equation (2) and choosing $\alpha = 70,000$ results in flange resistances of

$$R_F = \frac{70000 \times 20}{10400 \times 0.228} = 590 \text{ ohms}$$

and in stringer resistances of

$$R_L = \frac{70000 \times 20}{10400 \times 0.500} = 269 \text{ ohms}$$

As explained previously, the length of the first shoar bay
will be one-half of the remaining bays, or 10 and 20 inches,
respectively. From equation 4, the shear resistances will
therefore be for the first bay

$$r_1 = \frac{70000 \times 12}{4000 \times 0.024 \times 10} = 875 \text{ ohms}$$

and for each of the remaining bays

$$r_2 = \frac{70000 \times 12}{4000 \times 0.024 \times 20} = 437 \text{ ohms}$$

The input current I is chosen for convenience at 15 milliamperes;
this value corresponds to the load of 1000 pounds. Then by
equation (1)

$$\beta = \frac{15}{1000} = 0.015$$

These values of R, r, and β were set up on the computer
network and the corresponding currents were recorded as shown in
figure 9. Substitution of the current values in equations (1)
and (3) then gave the desired stress values. For instance, the
current in the first shoar resistance r_1 was 5.90 milliamperes
and by equation (3) the shear stress was

$$\tau = \frac{5.90}{0.015 \times 10 \times 0.024} = 1640 \text{ psi}$$

The results of the analysis are shown in the plots of figure 10.
Normal stresses derived by the computer method are shown as steps
for reasons previously explained, with the computed exact-method
curves superimposed. The exact-method curve of longitudinal stress
does not cross the last step exactly in the middle because of the
rapidly changing slope of the curve in this region. The plot for
shear stress by the computer method is shown as a set of "standpipes"
inasmuch as the transverse current corresponding to the shear can be
measured only at the transverse junction points. The maximum value
of stress given by this method does not agree very well with the
maximum value of the exact method.

The results can be improved by adjusting the length of the bays; that is, by using shorter lengths in the regions of steeper slope. In order to note the effect of this adjustment, the computer was set up with bay lengths of 50, 20, 15, 10, and 5 inches; and a new analysis was made. The results are shown in figure 11. The agreement with the theoretical curve is now excellent, both for axial stresses and for shear stresses.

Analysis of a Box Beam

When the cover of a box beam is to be analyzed, the resistances are determined and set up exactly as for the panel. The load, however, is now introduced into the flange by shear forces in the web. When the beam is loaded at the tip, the shear is uniformly distributed. In the network, however, it is, of course, necessary to introduce the current in discrete amounts at finite intervals. As for the previous case of the panel in which the shear resistances of the sheet were distributed, the current introduced at the free end of the box beam is made one-half of the amount introduced at succeeding points so as to obtain a series of steps which straddle the smooth curve.

As a numerical example the panel previously considered is used as the cover of a box beam. (See fig. 12.) Because the beam is symmetrical about the center line, only one-half of the beam need be considered. The depth h is 3 inches, and a load of 250 pounds is assumed on each side at the tip. Also, the beam is divided into five equal-length bays of 20 inches. The running shear per unit length of the flange is

$$\frac{P}{h} = \frac{250}{3} = 83.3 \text{ pounds per inch}$$

For a bay length of 20 inches the total shear force per bay is $83.3 \times 20 = 1666$ pounds. As before, a convenient value of 10 milliamperes is chosen for the current, and the constant β then is

$$\beta = \frac{10}{1666} = 0.0060$$

The equivalent network is shown in figure 13, and the corresponding stress-distribution plots are shown in figure 14. Since the slope of the exact-method curves does not change very

rapidly, the use of equal-length bays is shown to give very good agreement between calculated and test results. It should be noted that with the structure divided into equal bays with a half step at the loaded end, a half step is left unaccounted for at the root end. This omission may be disregarded without appreciable error.

ANALOGY FOR BOLTED-JOINT PROBLEMS

As shown in appendix A, the correspondence between the elements of a bolted joint (fig. 15) and the computer network (fig. 16) is given in the following table:

Bolted joint		Electrical network	
Quantity	Symbol	Quantity	Symbol
Axial load	P	Longitudinal current	I
Axial flexibility of plate	$2K_p$	Longitudinal resistance simulating plate	R_p
Axial flexibility of strap	K_s	Longitudinal resistance simulating strap	R_s
Bolt load	B	Transverse current	i
Bolt constant	C	Transverse resistance	r
Axial displacement	u	Potential	V

Written in equation form with arbitrary constants α and β, the following relations hold:

$$I = \beta P \tag{6}$$

$$i = \beta B \tag{7}$$

$$R_p = \alpha 2K_p \tag{8}$$

$$R_s = \alpha K_s \tag{9}$$

$$r = \alpha C \tag{10}$$

$$V = \alpha \beta u \tag{11}$$

From a comparison of figures 15 and 16, the resistor network will be noted to consist of a single row of longitudinal resistances R_s which simulate the two straps of the bolted joint. A single set of transverse resistances r_n which simulate the two shear areas of the bolt is also used. These apparent discrepancies are accounted for by the appearance of the factor 2 in equation (8).

When bolted-joint problems are to be solved, it should be noted that the load is transmitted through the bolts in discrete amounts acting at definite points, rather than through infinitesimal elements uniformly distributed along a sheet as in shear-lag problems. There is, therefore, no "finite-length increment" to cause errors when the network is used to simulate a bolted joint.

<center>NUMERICAL EXAMPLE OF A BOLTED-JOINT PROBLEM</center>

The bolted joint shown in figure 15 is considered with the following constants: for steel bolts,

$$d = 0.25 \text{ inch}$$

$$E_b = 29,000 \text{ ksi}$$

and for plates of 24S-T aluminun alloy,

$$t_p = 0.3125 \text{ inch}$$

$$t_s = 0.1875 \text{ inch}$$

$$p = 1.0 \text{ inch}$$

$$b = 2.0 \text{ inches}$$

$$E = 10,400 \text{ ksi}$$

The plate constants may be computed by the methods given in reference 3. From equation (2) of reference 3

$$K_s = \frac{p}{bt_s E} = \frac{1.0}{2 \times 0.1875 \times 10400} = 0.000256$$

and

$$2K_p = \frac{2p}{bt_p E} = \frac{2 \times 1.0}{2 \times 0.3125 \times 10400} = 0.000308$$

The bolt constant is given by equation (A19) of reference 3

$$C = \frac{8}{t_p E_b} \left\{ 0.13 \left(\frac{t_p}{D}\right)^2 \left[2.12 + \left(\frac{t_p}{D}\right)^2 \right] + 1.43 \right\}$$

When t_p is replaced by t_{av}, which is given by equation (A21) of reference 3, as

$$t_{av} = \frac{2t_s + t_p}{2} = \frac{2 \times 0.1875 + 0.3125}{2} = 0.344 \text{ inch}$$

the bolt constant becomes

$$C = \frac{8}{t_p E_b} \left\{ 0.13 \left(\frac{t_{av}}{D}\right)^2 \left[2.12 + \left(\frac{t_{av}}{D}\right)^2 \right] + 1.43 \right\}$$

$$= \frac{8}{0.3125 \times 29000} \left\{ 0.13 \left(\frac{0.344}{0.25}\right)^2 \left[2.12 + \left(\frac{0.344}{0.25}\right)^2 \right] + 1.43 \right\} = 0.00229$$

In setting up the resistance network, a convenient value for the constant α is 200,000; therefore, from equations (8), (9) and (10), respectively,

$$R_p = 200000 \times 0.000308 = 61.6 \text{ ohms}$$

$$R_s = 200000 \times 0.000256 = 51.2 \text{ ohms}$$

$$r = 200000 \times 0.00229 = 458.0 \text{ ohms}$$

These values were set up on the computer, and the transverse currents were measured and expressed as fractions of the total

applied current. For comparison, the corresponding bolt loads were computed by the methods of reference 3, and the results are tabulated as follows:

Bolt	Bolt load, B (fraction of total)	
	Analytical method	Electrical method
1	0.256	0.253
2	.185	.183
3	.159	.159
4	.173	.169
5	.228	.228

CONCLUSIONS

The electrical-network computer using available variable resistors with sources of current stabilized by simple electronic regulating circuits is a practical device for rapidly solving problems in shear lag and bolted joints. Sufficient accuracy is obtained to warrant the use of the computer in ordinary engineering stress analysis. The use of a more extended electrical network capable of simulating more complicated structures would appreciably reduce the time necessary for their analysis.

Langley Memorial Aeronautical Laboratory
    National Advisory Committee for Aeronautics
        Langley Field, Va. February 14, 1947

APPENDIX A

DEVELOPMENT OF ANALOGY BETWEEN LOADS IN BOLTED

JOINT AND CURRENT IN LADDER-TYPE NETWORK

Figure 16 shows a simple electrical ladder-type network set up to simulate a bolted joint. A current I is introduced into the network with part flowing down the line of resistances R_p and with part flowing through the transverse resistances r into the line of resistances R_s. The resistances R_p are all equal, as are R_s; but the resistances r are not necessarily equal.

Applying the principle that the voltage drop around any closed mesh must be zero results in

$$i_n r_n + I_{sn} R_s - i_{n+1} r_{n+1} - I_{pn} R_p = 0 \qquad (A1)$$

or

$$i_{n+1} r_{n+1} = i_n r_n + I_{sn} R_s - I_{pn} R_p \qquad (A2)$$

but

$$I_{sn} = \sum_{1}^{n} i_m \qquad (A3)$$

and

$$I_{pn} = I - I_{sn}$$

$$= I - \sum_{1}^{n} i_m \qquad (A4)$$

Substituting equations (A3) and (A4) in equation (A2) therefore gives

$$i_{n+1} \, r_{n+1} = i_n \, r_n - R_p \, I + (R_s + R_p) \sum_{1}^{n} i_m \qquad (A5)$$

Now

$$\sum_{1}^{n} i_m = i_n + \sum_{1}^{n-1} i_m \qquad (A6)$$

Substituting equation (A6) in equation (A5) and rearranging terms results in

$$i_{n+1} = \frac{r_n}{r_{n+1}} \, i_n + \frac{R_s + R_p}{r_{n+1}} \, i_n - \frac{R_p}{r_{n+1}} \, I + \frac{R_s + R_p}{r_{n+1}} \sum_{i}^{n-1} i_m \qquad (A7)$$

From equation (1) of reference 3 the corresponding mechanical equation is

$$B_{n+1} = \frac{C_n}{C_{n+1}} \, B_n + \frac{2K_p + K_s}{C_{n+1}} \, B_n - \frac{2K_p}{C_{n+1}} \, P + \frac{2K_p + K_s}{C_{n+1}} \sum_{i}^{n-1} B_m \qquad (A8)$$

The following analogy is therefore obtained:

Bolted joint		Electrical network	
Quantity	Symbol	Quantity	Symbol
Axial load	P	Longitudinal current	I
Axial flexibility of plate	$2K_p$	Longitudinal resistance simulating plate	R_p
Axial flexibility of strap	K_s	Longitudinal resistance simulating strap	R_s
Bolt load	B	Transverse current	i
Bolt constant	C	Transverse resistance	r
Axial displacement	u	Potential	V

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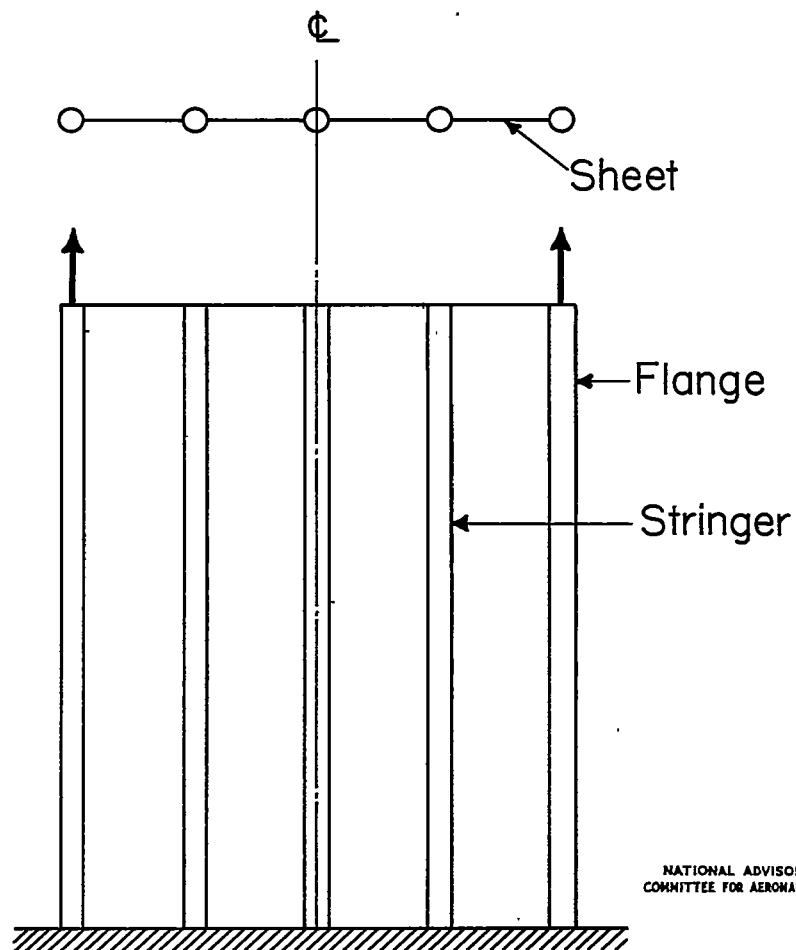
Figure 1.- Simple stiffened panel.

Figure 2.- Elementary bolted joint.
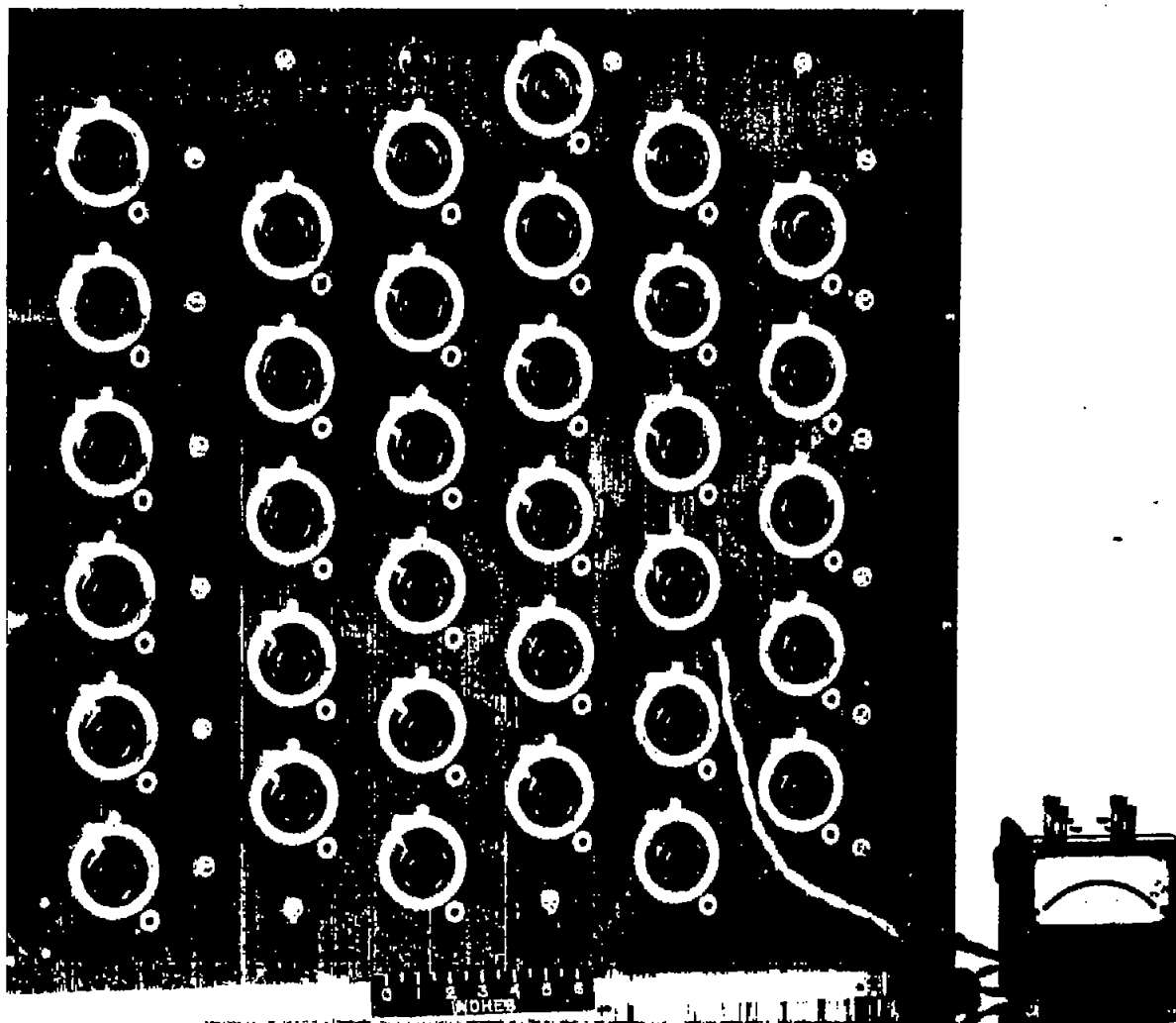
Figure 3.- Front panel of electrical computer used in solution of shear-lag and bolted-joint problems.

Fig. 3
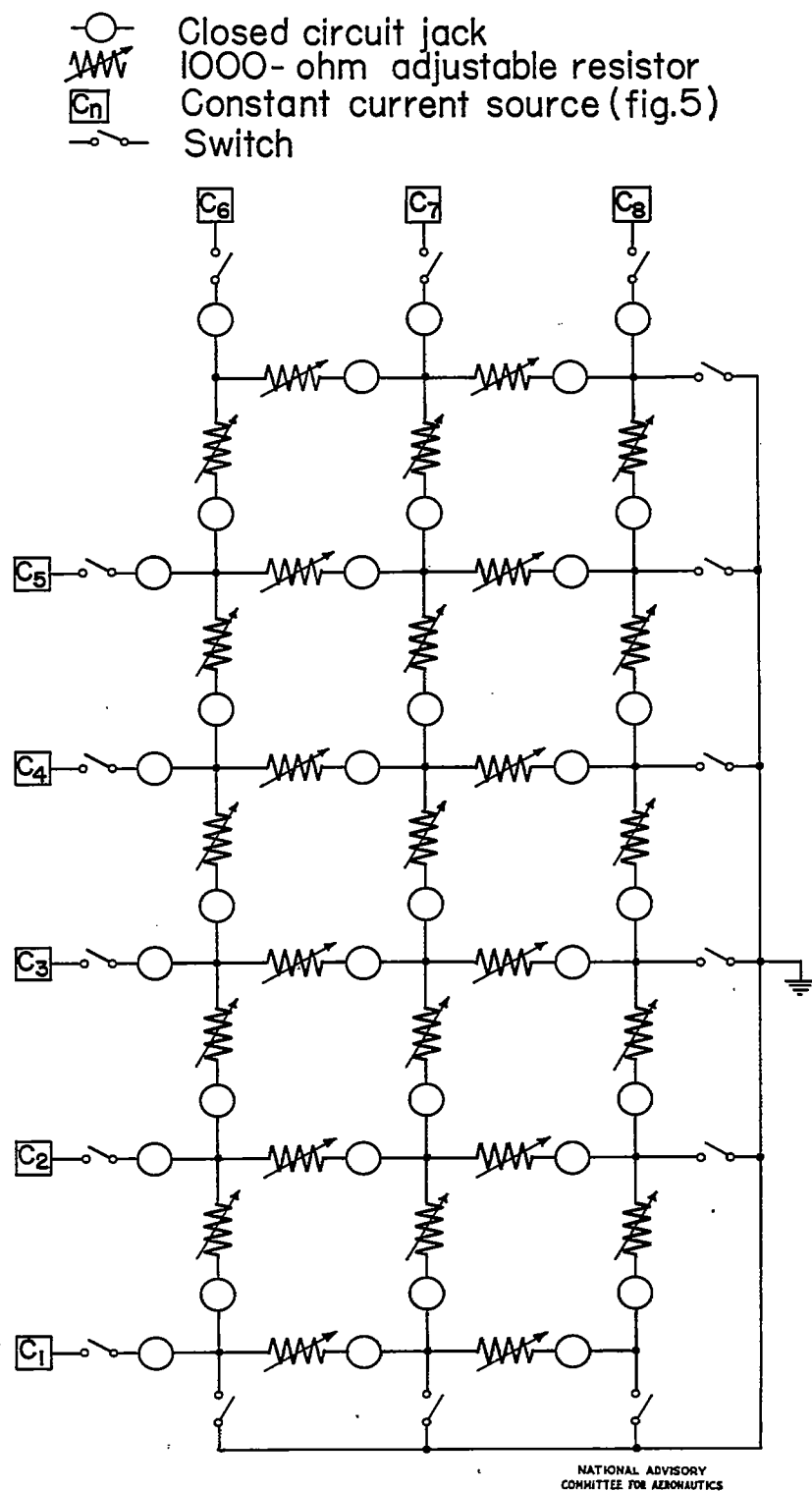
Figure 4.- Wiring diagram of network.

Fig. 5

15,000
ohms

To network

Vacuum tube
type 6F6

400-volt
d-c. supply

Vacuum tube
type VR 150

500 ohms
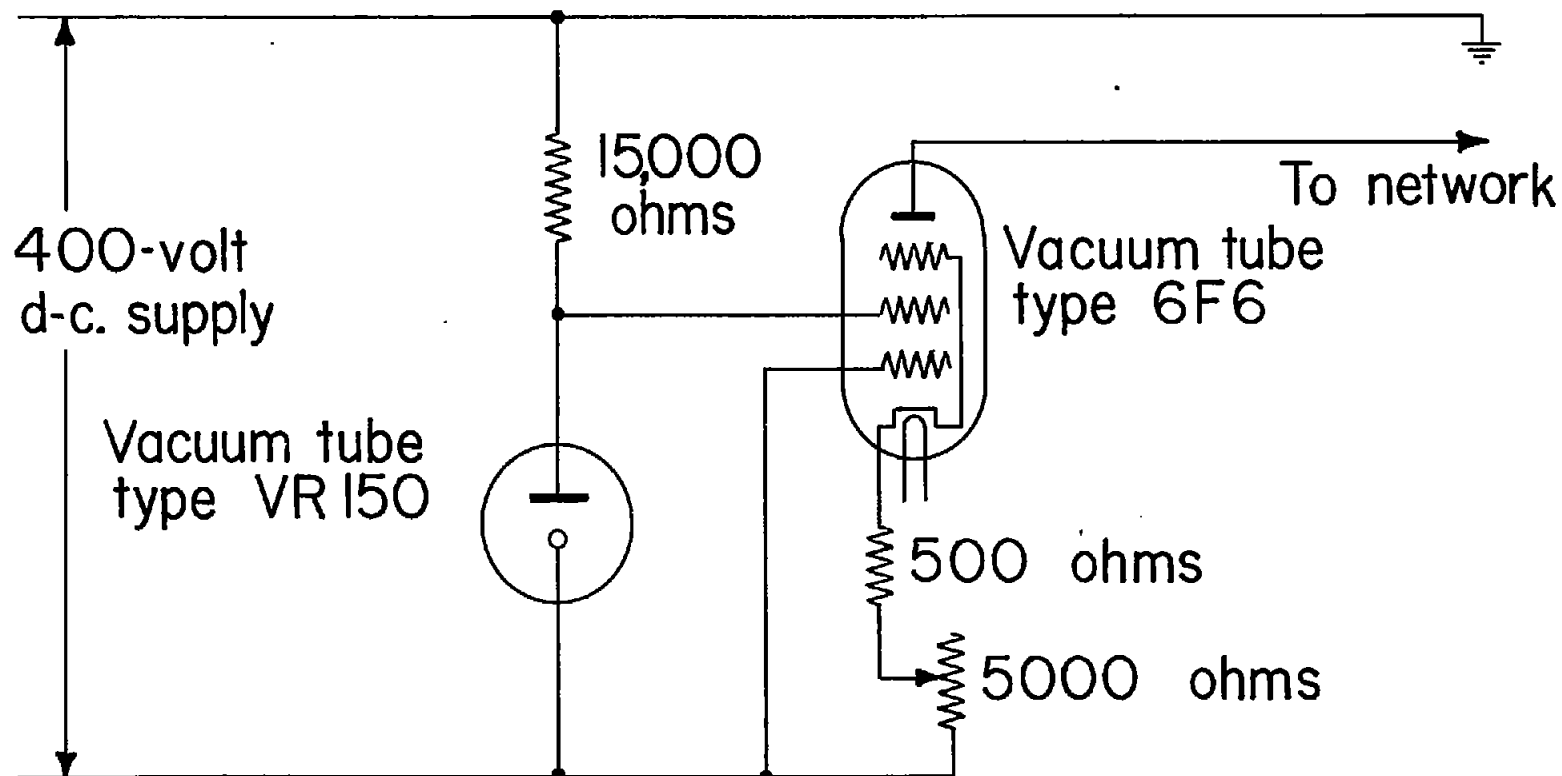
5000 ohms

Figure 5.- Wiring diagram of one constant
current source.

NACA LMAL 44254
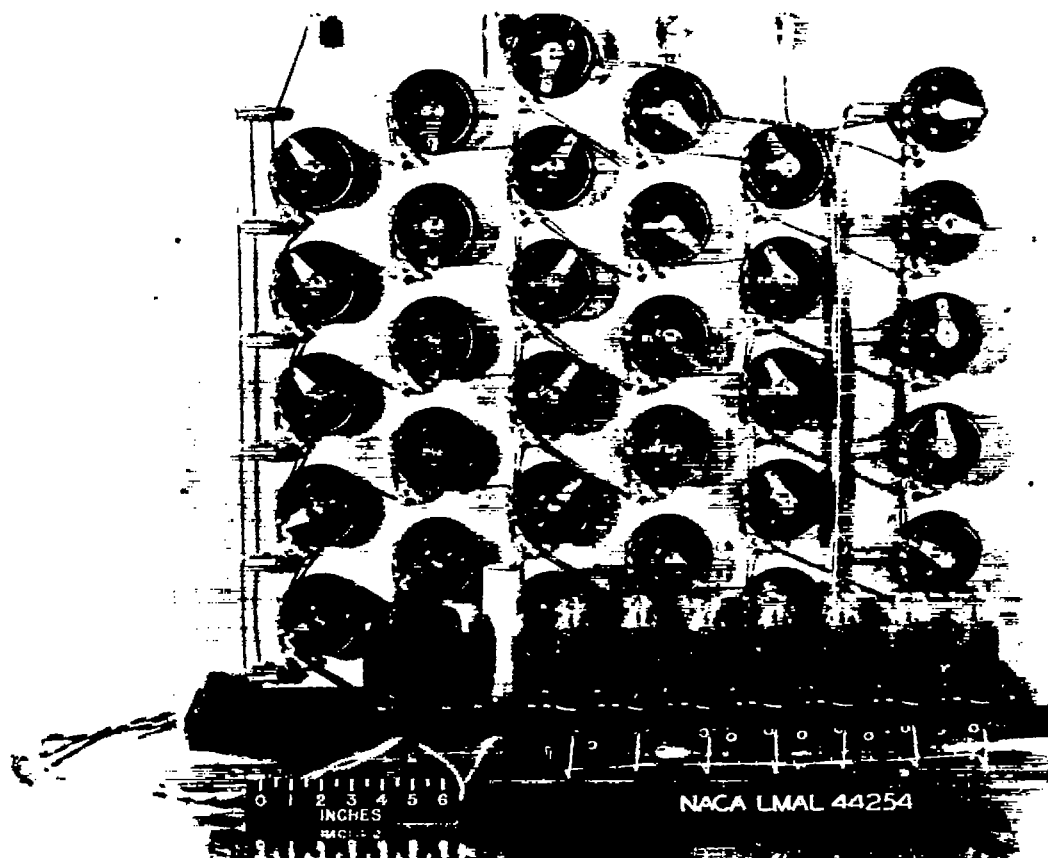
Figure 6.- Rear view of panel of electrical computer used in solution of shear-lag and bolted-joint problems.
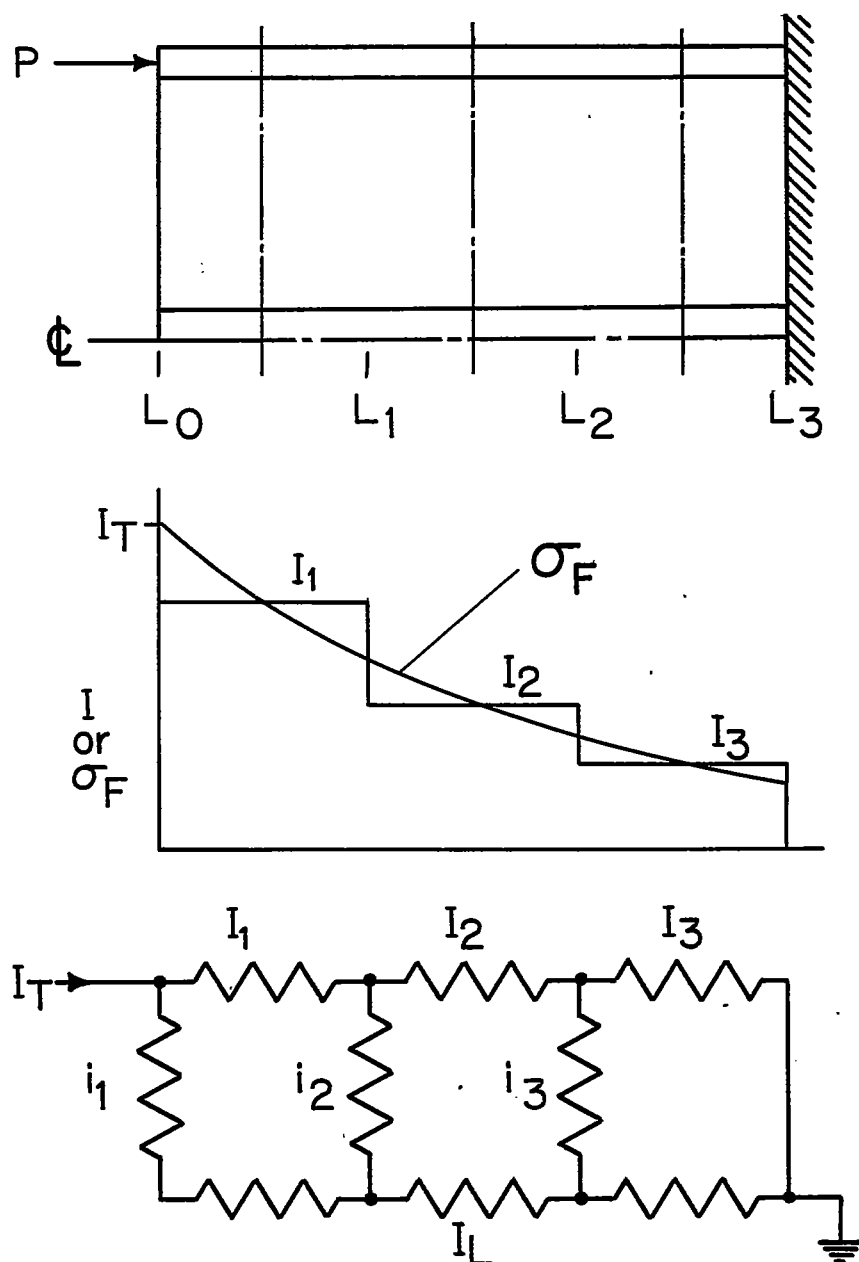
Figure 7.- Stiffened panel and equivalent network.

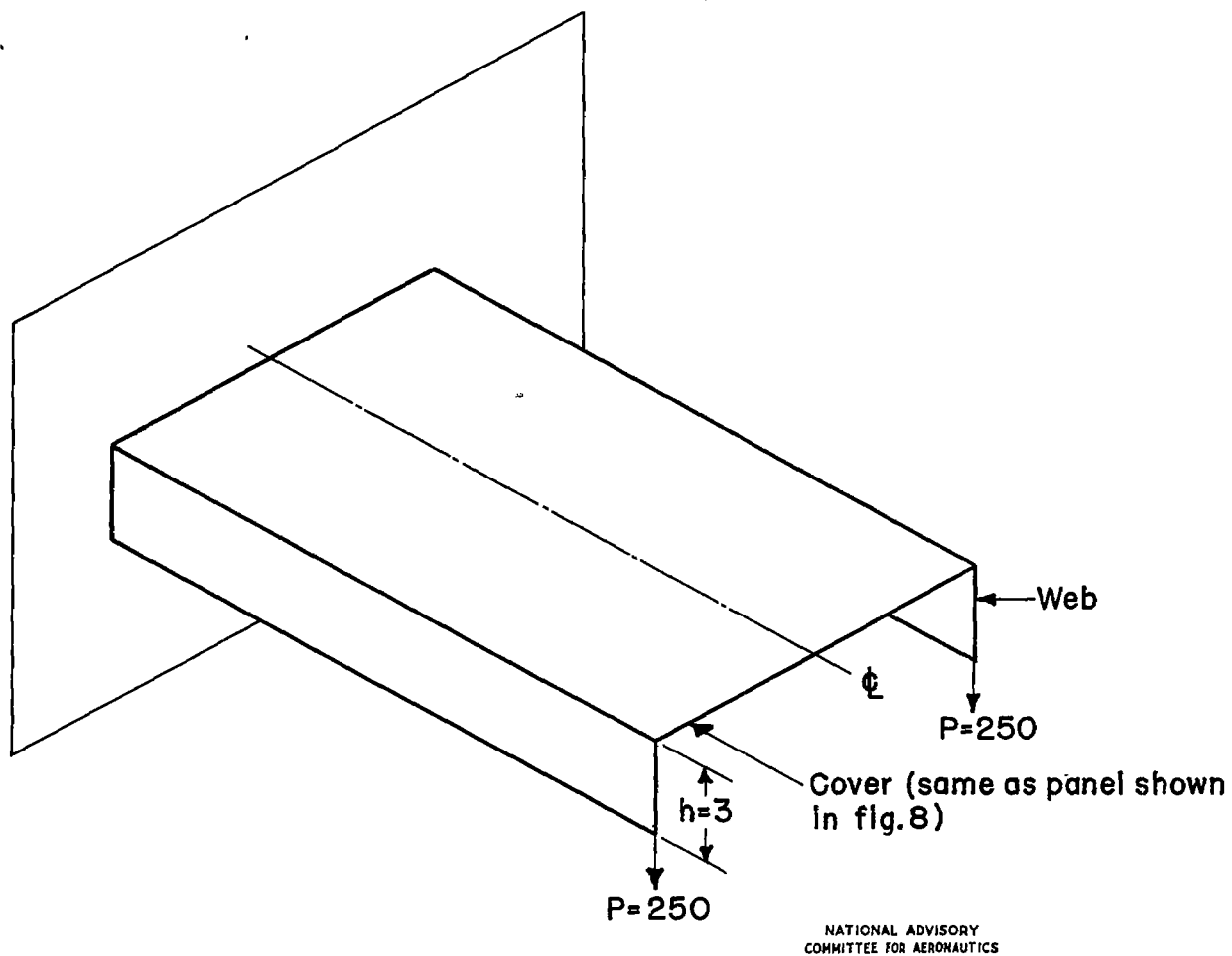Fig. 8   NACA TN No. 1281
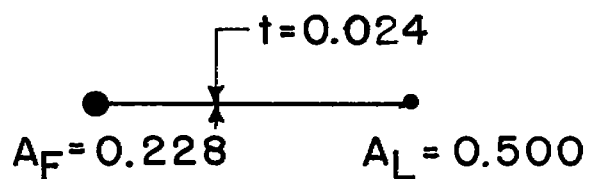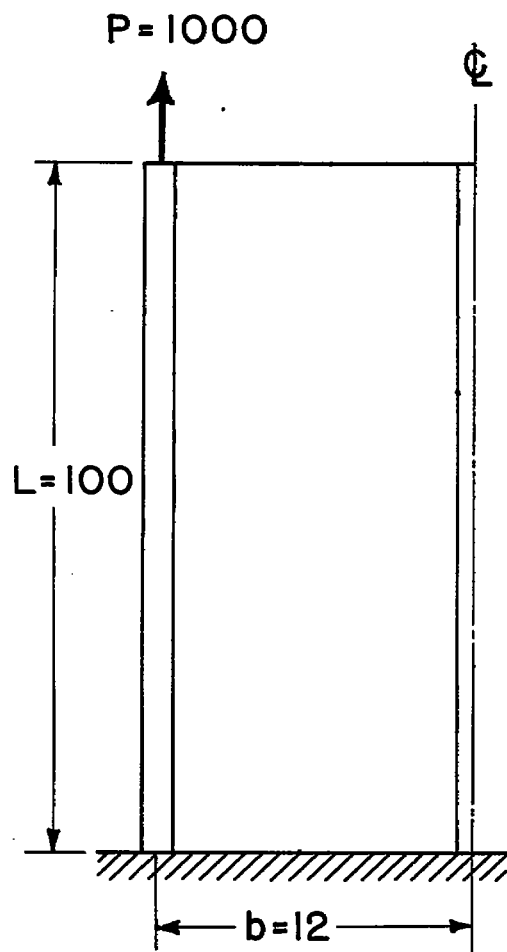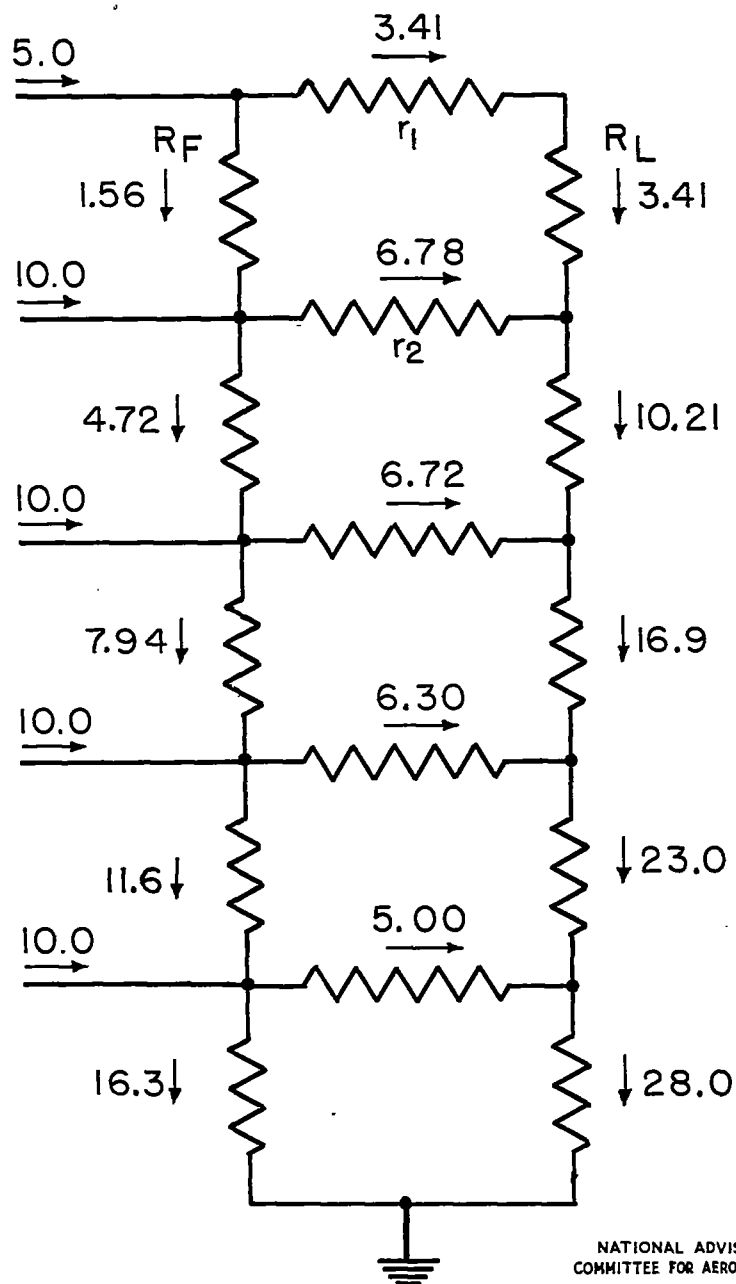

Figure 8.– Stiffened panel for numerical example.
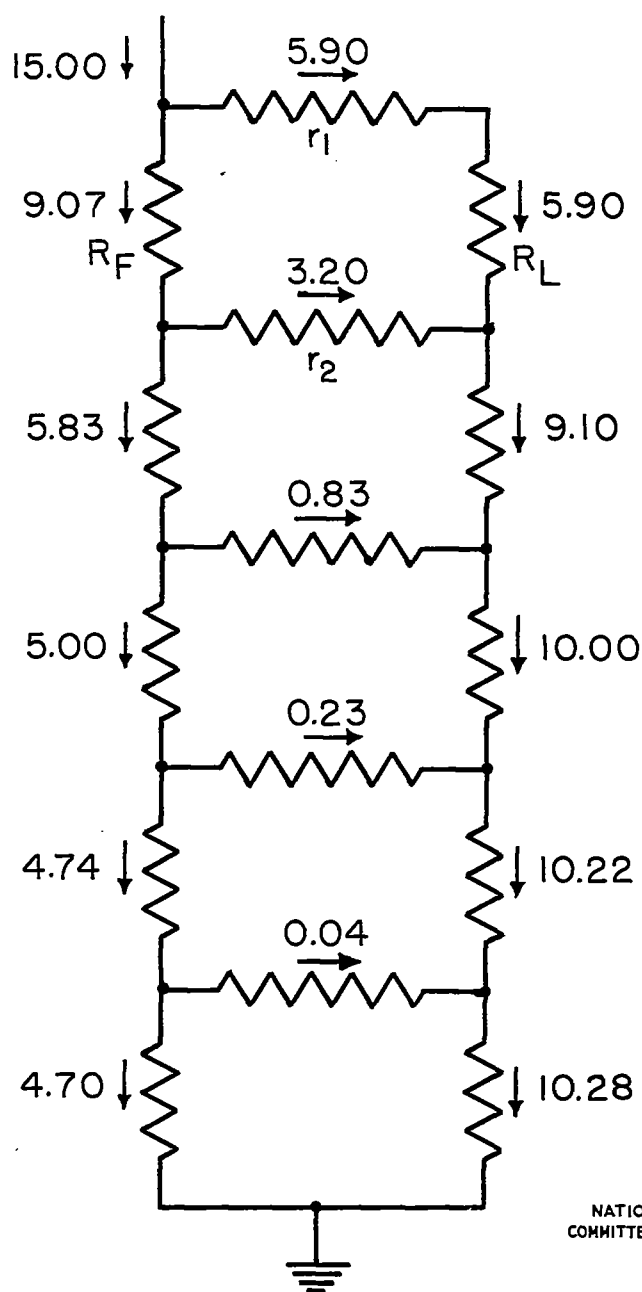
Figure 9.- Equivalent network for panel showing magnitude and direction of current. $R_F = 590$ ohms; $R_L = 269$ ohms; $r_1 = 875$ ohms; $r_2 = 437$ ohms.

Fig. 10   NACA TN No. 1281
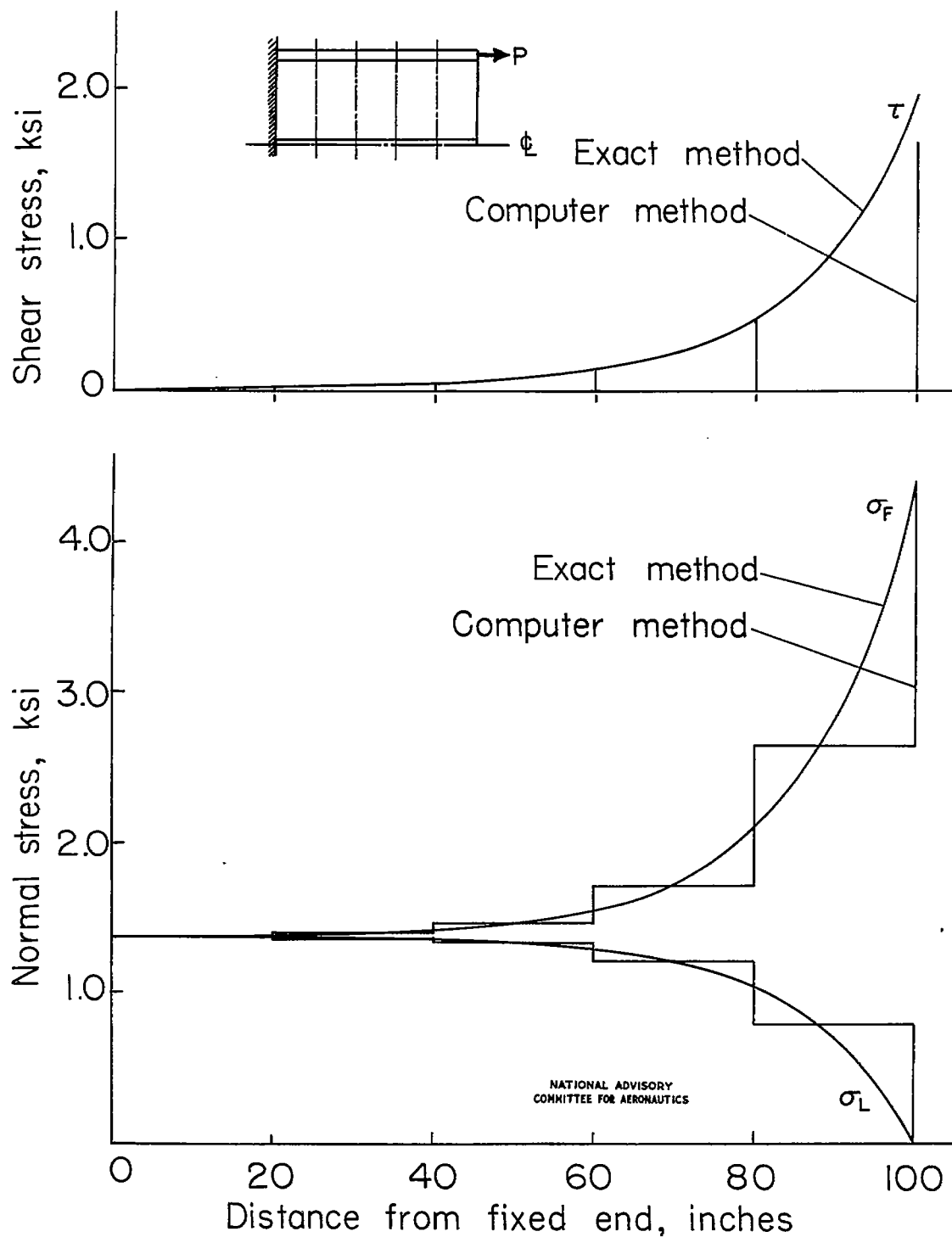
Figure 10.- Stresses in panel of equal bays.
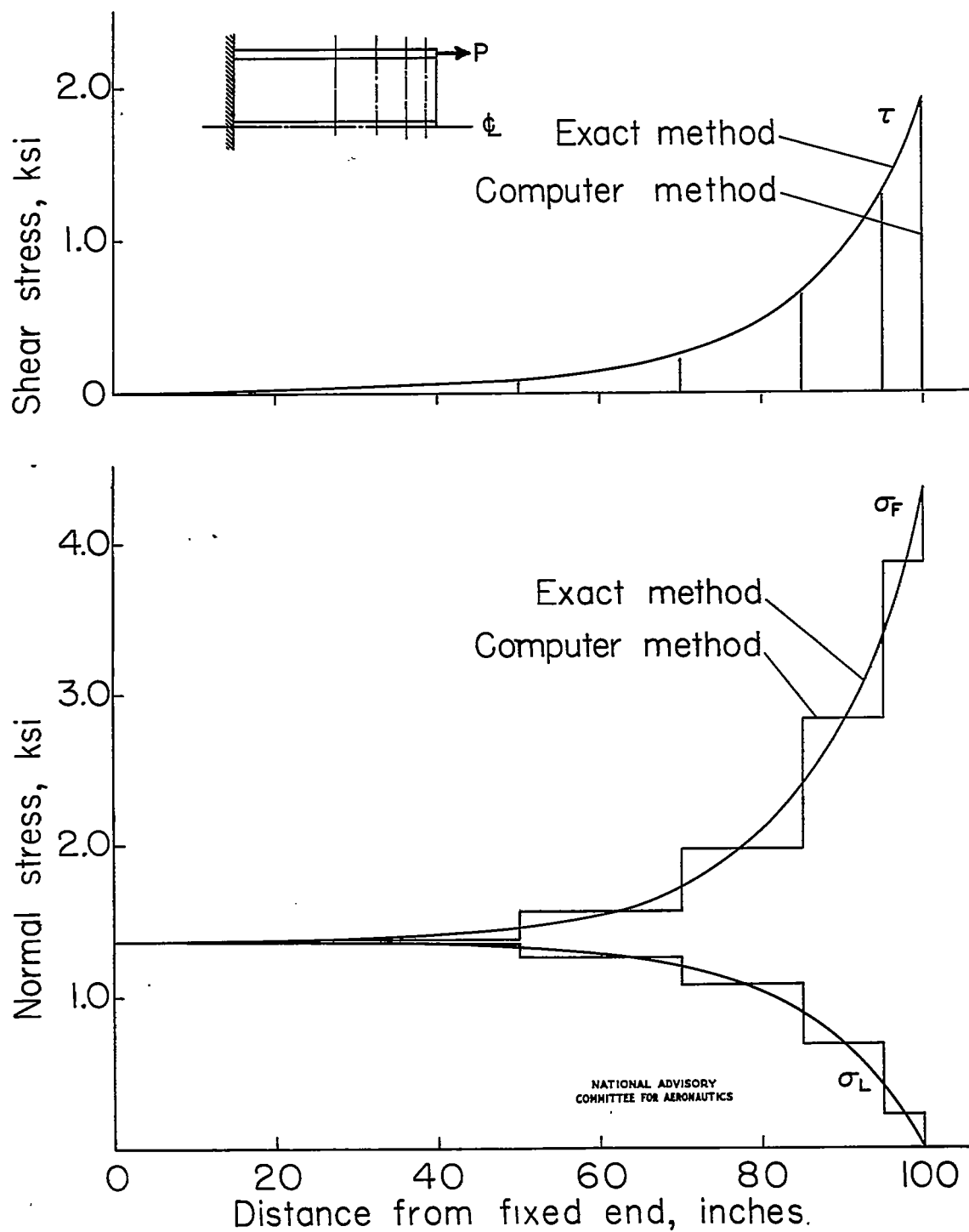
Figure 11.- Stresses in panel of unequal bays.

Fig. 12

NACA TN No. 1281

Web

P=250

Cover (same as panel shown
in fig.8)

h=3

P=250

¢

Figure 12.- Box beam for numerical example.

Figure 13.- Equivalent network for box beam showing magnitude and direction of current. $R_F = 590$ ohms; $R_L = 269$ ohms; $r_1 = 875$ ohms; $r_2 = 437$ ohms.
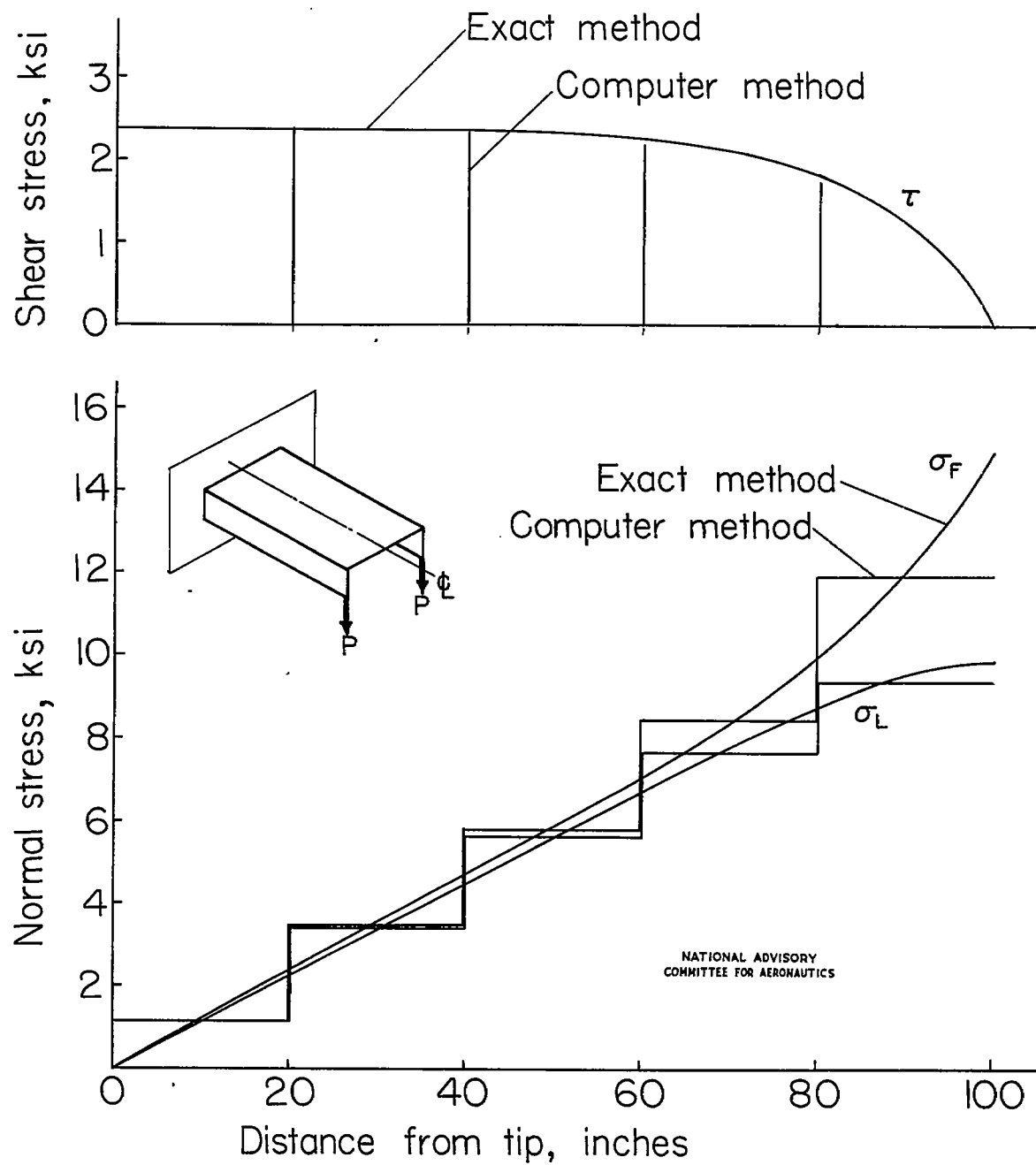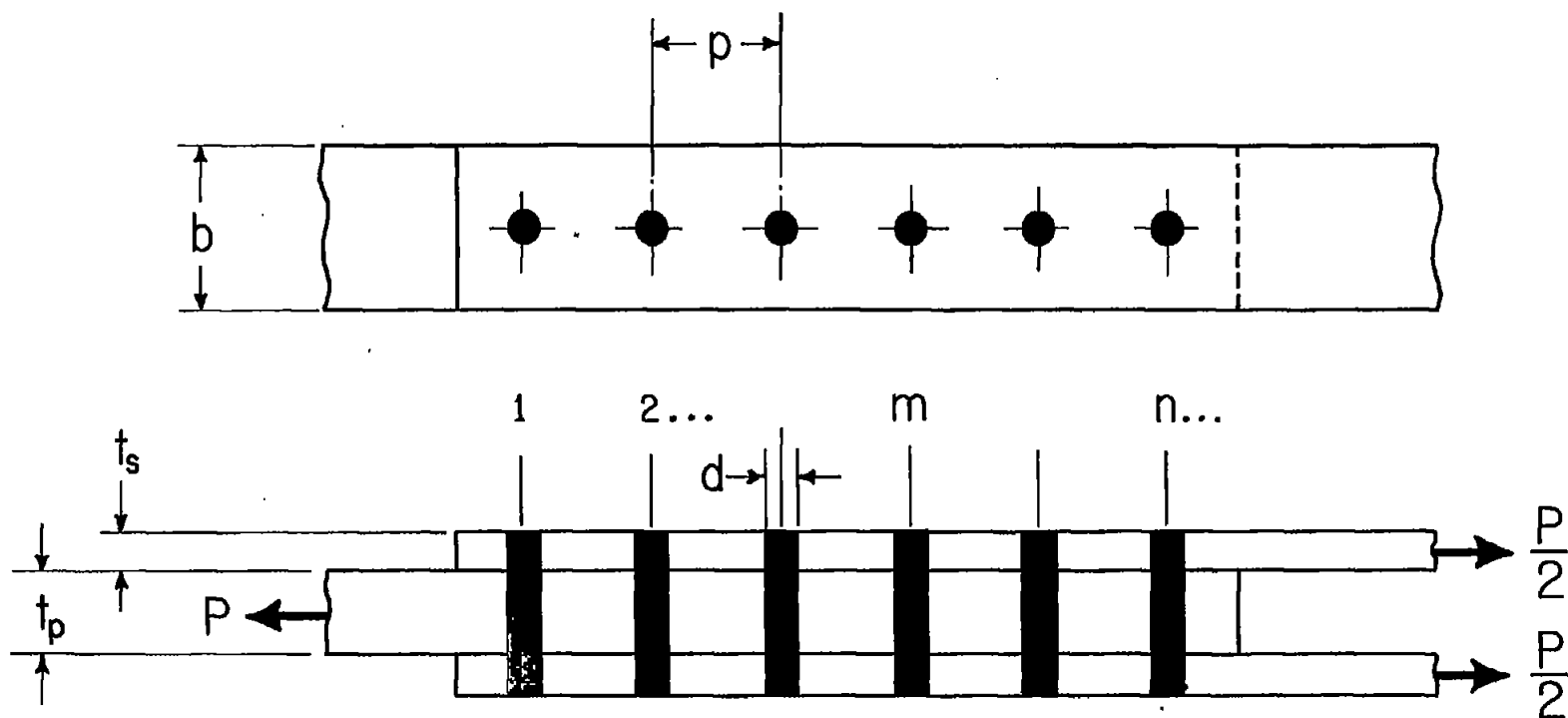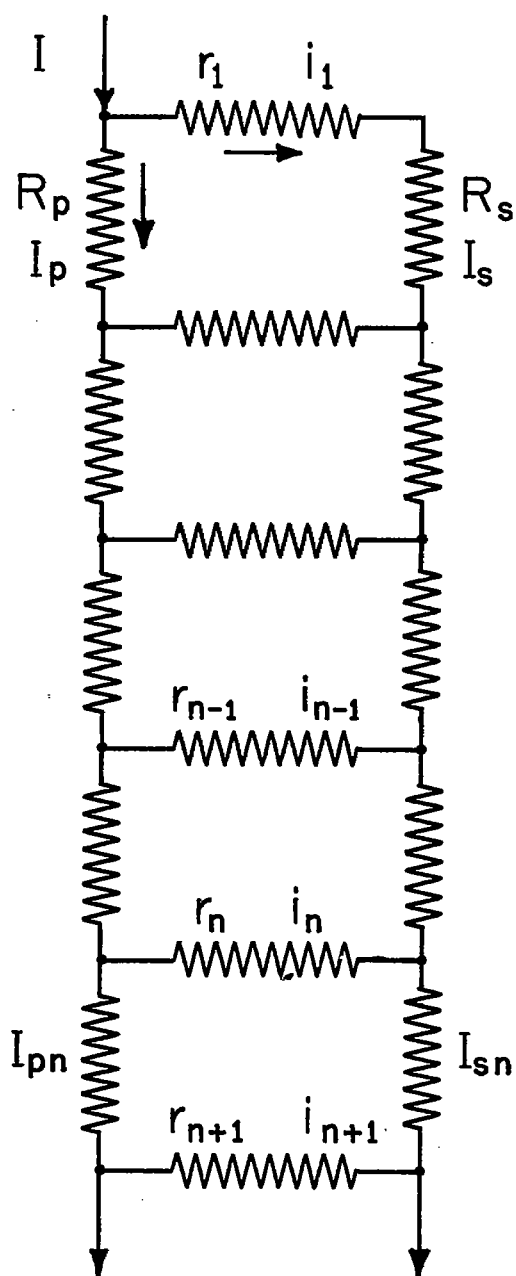
Fig. 14   NACA TN No. 1281

Figure 14.- Stresses in beam cover.

Figure 15.- Typical bolted joint.

Fig. 15

Fig. 16   NACA TN No. 1281

Figure 16.- Equivalent network for
bolted joint.